Introduction to Probability

Topics : Computer engineering
Written on March 13, 2024

1. **Definition:**
   - Probability is a measure of the likelihood that an event will occur.
   - It is expressed as a number between 0 and 1, where 0 indicates impossibility and 1 indicates certainty.
2. **Sample Space and Events:**
   - The sample space, denoted by S, is the set of all possible outcomes of an experiment.
   - An event is any subset of the sample space.
3. **Probability of an Event:**
   - The probability of an event A, denoted by P(A), is the sum of the probabilities of all outcomes in A.
   - It satisfies the following properties:
     1. $0 \leq P(A) \leq 1$ for any event A.
     2. $P(S) = 1$, where S is the sample space.
     3. If A and B are disjoint events (i.e., they have no outcomes in common), then $P(A \cup B) = P(A) + P(B)$.
4. **Probability Rules:**
   - **Complement Rule:** The probability of the complement of an event A, denoted by A' or A^c, is $P(A') = 1 - P(A)$.
   - **Union Rule:** The probability of the union of two events A and B, denoted by $A \cup B$, is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
   - **Intersection Rule:** If A and B are independent events, then $P(A \cap B) = P(A) \times P(B)$.
5. **Types of Probability:**
   - **Classical Probability:** Based on equally likely outcomes in a sample space.
   - **Empirical Probability:** Based on observed frequencies from data.
   - **Subjective Probability:** Based on personal judgment or opinion.
6. **Conditional Probability:**
   - Conditional probability measures the likelihood of an event occurring given that another event has already occurred.
   - It is denoted by $P(A|B)$ and calculated as $P(A|B) = P(A \cap B) / P(B)$.

7. **Independence:**
    - Two events A and B are independent if the occurrence of one event does not affect the occurrence of the other.
    - Mathematically, $P(A \cap B) = P(A) \times P(B)$.