Complex Numbers

Topics : Computer engineering
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1. **Definition:**
    - A complex number is a number that can be expressed in the form a + bi, where a and b are real numbers, and i is the imaginary unit with the property i^2 = -1.
    - The real part of the complex number is a, and the imaginary part is bi.
2. **Operations:**
    - **Addition and Subtraction:** Complex numbers are added or subtracted by adding or subtracting their real and imaginary parts separately.
    - **Multiplication:** To multiply complex numbers (a + bi) and (c + di), distribute and combine like terms, then simplify using i^2 = -1.
    - **Division:** To divide complex numbers, multiply the numerator and denominator by the complex conjugate of the denominator, then simplify.
3. **Complex Conjugate:**
    - The complex conjugate of a complex number a + bi is a - bi.
    - When multiplied together, a complex number and its conjugate yield a real number: (a + bi)(a - bi) = a^2 + b^2.
4. **Polar Form:**
    - Complex numbers can also be represented in polar form as r(cos θ + i sin θ), where r is the magnitude (or modulus) of the complex number and θ is the argument (or angle) measured counterclockwise from the positive real axis.
    - The magnitude of a complex number a + bi is √(a^2 + b^2), and the argument is arctan(b/a).
5. **Euler's Formula:**
    - Euler's formula relates complex numbers, trigonometric functions, and the exponential function: e^(iθ) = cos θ + i sin θ.
    - This formula allows complex numbers to be expressed in terms of exponentials, facilitating operations like exponentiation and finding roots.
6. **Applications:**
    - Complex numbers have applications in various fields, including electrical engineering, signal processing, quantum mechanics, and fluid dynamics.

- They are used to represent alternating current (AC) circuits, analyze oscillatory motion, solve differential equations, and more.